WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST

SEMESTER TWO 2019

TEST 5: Differentiation and Differential Equations

Name: _____

Friday 30th August 2019

Time: 55 minutes

Total marks: ___ + ___ = ___
20 30 50

Calculator free section – maximum 25 minutes

1. [6 marks]

   An electrical device, subject to a constant voltage of 24 Volts, has a resistance R that is decreasing at a rate of 0.1 Ohm per second.

   (An Ohm is the standard unit of electrical resistance.)

   The voltage V, current I (in Ampere) and resistance R follow Ohm's Law: $V = I \times R$

   Describe (quantitatively) how the current is changing when the resistance is 4.0 Ohm.

2. [9 marks – 2, 4 and 3]

(a) A particle is travelling in a straight line with velocity v related to displacement x by the equation: $v = 2\sqrt{x-1}$. Show that acceleration a is a constant

(b) For $v = 2\sqrt{x-1}$, determine x as a function of time t, if $x(t=0) = 5$

(c) If acceleration $a = \cos x$, find v in terms of x when $v\left(x = \frac{\pi}{2}\right) = 2$ and $v \geq 0$.

3. [5 marks – 4 and 1]

A population of bacteria, P at time t, is growing at a rate modelled by:

$$\frac{dP}{dt} = P - \frac{P^2}{1000}$$

(a) Show, by differentiation (and substitution), that $P = \frac{1000}{1+Ce^{-t}}$ satisfies this differential equation, for any value of the constant C.

(b) Calculate C if $P(t=0) = 10$

Year 12 Specialist Test 5: Derivatives and Differential Equations

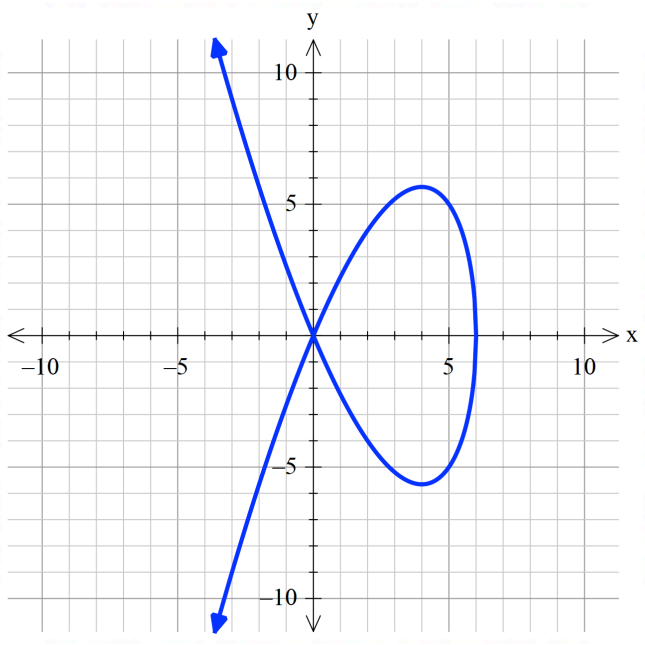
Name: _____

Time: 35 minutes 30 marks

Calculator assumed section

4. [7 marks – 2, 2 and 3]

The curve defined by $y^2 = x^2(6-x)$, as shown, is another right strophoid.



(a) Derive an expression for $\frac{dy}{dx}$ in terms of both x and y.

(b) Determine the exact co-ordinates of the relative minimum and maximum points on the closed part of the curve.

(c) Investigate the value(s) of the slope of the curve at the origin. The graph shows that these slopes are defined!

5. [10 marks – 1, 2, 2, 2, 1, 1 and 1]

The individual seat bookings, B, for a school production are increasing at a rate modelled by

$$\frac{dB}{dt} = kB(3800 - B)$$

(a) What is the maximum number who might attend this production?

At the instant when 80 bookings had been made, bookings were increasing at a rate of 50 per day.

(b) Show clearly that $k = \frac{1}{5952}$

(c) What is the maximum rate of increase of bookings?

After three days, 256 seats had been booked.

(d) Write an equation to represent the number of bookings as a function of t.

Question 5 (continued)

Determine the:

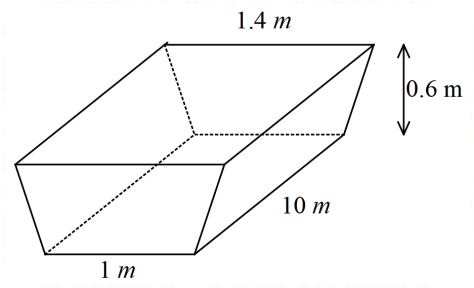
(e) initial number of bookings

(f) number of bookings made in the first 8 days

(g) day on which bookings close because only 100 seats remain unsold.

6. [4 marks]

A water trough 10 m long has a trapezoidal cross-section as shown.



It is being filled at a rate of 60 litres per minute.

When the water is h m deep, the volume V (in m^3) is given by $V = 10\left(h + \frac{h^2}{3}\right)$

How fast is the water level rising:

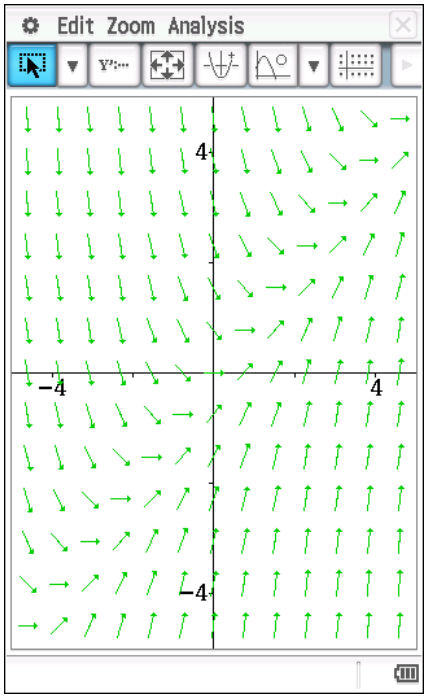
(a) initially (when $h = 0$)

(b) when $h = 0.3$ m

7. [9 marks – 1, 2, 1, 2 and 3]

Slope or gradient fields enable us to analyse differential equations that are difficult to solve.

Consider $\frac{dy}{dx} = x - y$, as shown.



(a) Describe the locus of points with a horizontal gradient.

(b) Sketch the solution to $\frac{dy}{dx} = x - y$ that passes through $(-3,1)$

(c) Sketch the solution to $\frac{dy}{dx} = x - y$ that passes through $(3,-1)$

(d) Describe and generalise the differences between these solutions in (b) and (c)

(e) Use Euler's method, with $\delta x = 0.1$ to estimate $y(x = 2.4)$ for the solution that passes through the point $(2,2)$
